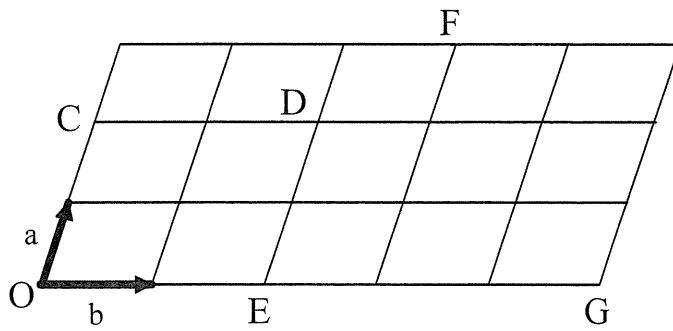


3. (7 marks)



Determine expressions for each of the following in terms of **a** and/or **b** as shown in the diagram.

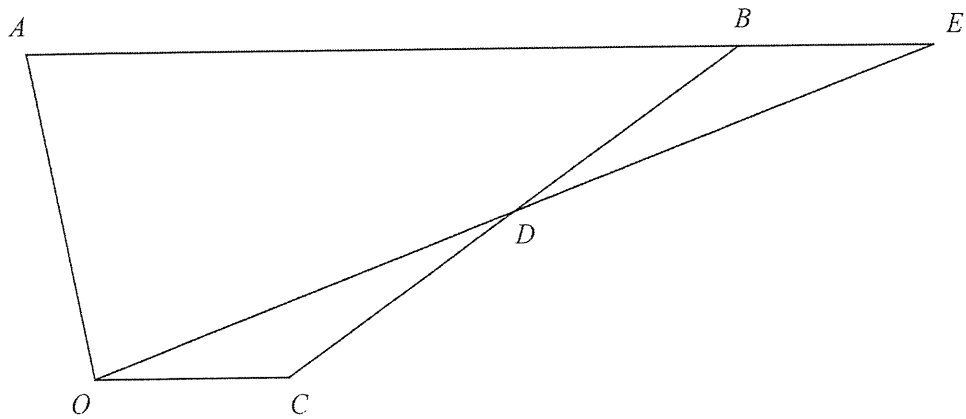
(a) \overline{OC} $\quad 2\underset{\sim}{a}$ [1]

(b) \overline{OF} $\quad 3\underset{\sim}{b} - 2\underset{\sim}{a}$ [2]

(c) \overline{CE} $\quad 2\underset{\sim}{b} - 2\underset{\sim}{a}$ [2]

(d) \overline{GD} $\quad 2\underset{\sim}{a} - 3\underset{\sim}{b}$ [2]

4. (7 marks)



For the trapezium $OABC$ above, $\overrightarrow{OA} = \underline{a}$, $\overrightarrow{OC} = \underline{c}$, $\overrightarrow{AB} = 3\underline{c}$, and $\overrightarrow{CD} = \frac{1}{2}\overrightarrow{CB}$.

If $\overrightarrow{OE} = h\overrightarrow{OD}$ and $\overrightarrow{AE} = k\overrightarrow{AB}$, determine the value of h and k .

$$\begin{aligned}\overrightarrow{OE} &= h\overrightarrow{OD} \\ &= h\left(\frac{\underline{a}}{2} + 2\underline{c}\right)\end{aligned}$$

$$\begin{aligned}\overrightarrow{OE} &= \underline{a} + k\overrightarrow{AB} \\ &= \underline{a} + k3\underline{c}\end{aligned}$$

$$\frac{ha}{2} + 2ch = \underline{a} + 3kc$$

$$\frac{ha}{2} - \underline{a} = 3kc - 2ch$$

$$\underline{a}\left(\frac{h}{2} - 1\right) = c(3k - 2h)$$

$$\frac{h}{2} - 1 = 0$$

$$h = 2$$

$$3k - 2h = 0$$

$$3k - 4 = 0$$

$$k = \frac{4}{3}$$

Mathematics Specialist Units 1,2
Test 1 2017

Section 2 Calculator Assumed
Counting, Basic Vectors

STUDENT'S NAME SOLNS

DATE: Friday 3 March

TIME: 40 minutes

MARKS: 40

INSTRUCTIONS:

Standard Items: Pens, pencils, drawing templates, eraser

Special Items: Three calculators, notes on one side of a single A4 page (these notes to be handed in with this assessment)

Questions or parts of questions worth more than 2 marks require working to be shown to receive full marks.

5. (10 marks)

A committee consists of 5 women and 7 men. Sub-committees must be formed from this group. How many different sub-committees of 5 can be formed in each situation below.

(a) no restrictions $\binom{12}{5} = 792$ [1]

(b) if the president must be included $\binom{1}{1} \binom{11}{4} = 330$ [2]

(c) two men refuse to be on the same committee $\binom{2}{0} \binom{10}{5} + \binom{2}{1} \binom{10}{4} = 672$ [3]

(d) one woman will only serve on the committee if one particular man is also on the committee [4]

$$\binom{2}{2} \binom{10}{3} + \binom{1}{1} \binom{1}{0} \binom{10}{4} + \binom{2}{0} \binom{10}{5}$$

$$= 582$$

6. (11 marks)

Using only the digits 2, 3, 4, 5, 6 and 7, without repetition, how many different numbers

(a) have 4 digits

[1]

$$6P4 = 360$$

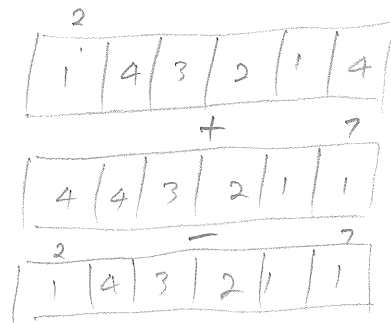
(b) start with a 2

[1]

$$1 \times 5! = 120$$

(c) start with a 2 or end with a 7

[3]



$$2 \times 4 \times 4! - 4! = 216$$

(d) are less than 600

[3]

$$\boxed{6} + \boxed{6 \mid 5} + \boxed{4 \mid 5 \mid 4} = 480$$

(e) are less than 600 and even

[3]

$$\boxed{3} + \boxed{5 \mid 3} + \boxed{4 \mid 4 \mid 1} + \boxed{7 \mid 4 \mid 2} = 58$$

7. (4 marks)

Determine the value of λ and μ if $3\lambda\mathbf{a} + \mathbf{a} + 4\mu\mathbf{b} = \mathbf{b} - 2\mu\mathbf{a} - 7\lambda\mathbf{b}$ where \mathbf{a} and \mathbf{b} are non-parallel vectors.

$$\begin{aligned}
 3\lambda\mathbf{a} + \mathbf{a} + 4\mu\mathbf{b} &= \mathbf{b} - 7\lambda\mathbf{b} - 4\mu\mathbf{b} \\
 \underline{2(3\lambda + 1 + 2\mu)} &= \underline{2(1 - 7\lambda - 4\mu)} \\
 3\lambda + 1 + 2\mu &= 0 \\
 1 - 7\lambda - 4\mu &= 0 \\
 6\lambda + 2 + 4\mu &= 0 \\
 + \quad 1 - 7\lambda - 4\mu &= 0 \\
 \hline
 -\lambda + 3 &= 0 \\
 \lambda &= 3 \\
 \mu &= -5
 \end{aligned}$$

8. (4 marks)

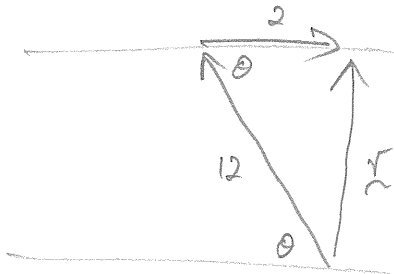
Prove $\binom{n}{r} + \binom{n}{r+1} = \binom{n+1}{r+1}$

$$\begin{aligned}
 \text{LHS} &= \frac{n!}{r!(n-r)!} + \frac{n!}{(r+1)!(n-r-1)!} \\
 &= \frac{n!(r+1)}{r!(n-r)!(r+1)} + \frac{n!(n-r)}{(r+1)!(n-r-1)!(n-r)} \\
 &= \frac{n!(r+1)}{(r+1)!(n-r)!} + \frac{n!(n-r)}{(r+1)!(n-r)!} \\
 &= \frac{\cancel{r} \times n! + n! + n \times n! - \cancel{r} \times n!}{(r+1)!(n-r)!} \\
 &= \frac{n!(n+1)}{(r+1)!(n-r)!} \\
 &= \frac{(n+1)!}{(r+1)!(n-r)!} \\
 &= \binom{n+1}{r+1} = \text{RHS}
 \end{aligned}$$

9. (6 marks)

A boat has a speed of 12 km/hr in still water. It is to be driven so that it travels directly across a river 175 metres wide. The river is flowing at 2 km/hr.

(a) At what angle to the bank should the boat be steered? [3]

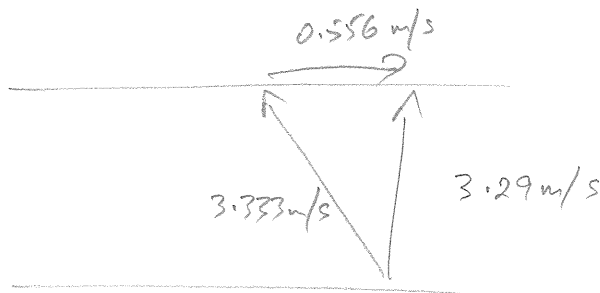


$$\cos \theta = \frac{2}{12}$$

$$\theta = 80.4^\circ$$

(b) How long will it take to reach the other side? [3]

$$\text{TIME} = \frac{\text{DIST}}{\text{SPEED}}$$



$$\begin{aligned} \text{TIME} &= \frac{175}{3.29} \\ &= 53.2 \text{ sec} \end{aligned}$$

10. (5 marks)

Calculate the magnitude of force P and the size of θ if the three forces shown in the diagram are in equilibrium.

